

Sparse Signals Recovery from Noisy Measurements by Orthogonal Matching Pursuit ^{*†}

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Abstract

Recently, many practical algorithms have been proposed to recover the sparse signal from fewer measurements. Orthogonal matching pursuit (OMP) is one of the most effective algorithm. In this paper, we use the restricted isometry property to analysis the algorithm. We show that, under certain conditions based on the restricted isometry property and the signals, OMP will recover the support of the sparse signal when measurements are corrupted by additive noise.

1 Introduction

Compressed sensing shows that it is high possibility to reconstruct sparse signals from their projection onto a small number of random vectors, possibly corrupted by noise. Let $\|\mathbf{x}\|_0$ denote the number of nonzero entries of vector \mathbf{x} . If $\|\mathbf{x}\|_0 < K$, a signal \mathbf{x} is said to be K -sparse. Let A be an $m \times n$ measurement matrix with $m < n$. In compressed sensing, we are interested in recovering the K -sparse signal \mathbf{x} from

$$\mathbf{y} = A\mathbf{x} + \mathbf{z}, \quad (1.1)$$

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where \mathbf{z} is the noise term. Then, the approach would be to solve the following l_0 minimization problem:

$$\min_x \|A\mathbf{x} - \mathbf{y}\|_2 \quad \text{subject to} \quad \|\mathbf{x}\|_0 < K. \quad (1.2)$$

A greedy algorithm named orthogonal matching pursuit (OMP) is one of the efficient approach to solve (1.2). The basic idea of this iterative algorithm is to find the support of the unknown signal. At each iteration, one column of A that is the most correlated with the residue is selected. Then the residue is updated by projecting y onto the linear subspace spanned by the columns that have been selected. Basic reference for this method are [7, 14] and [16]. There are several natural stopping criteria for OMP [17]. Let \mathbf{r}_k be the residual in the each iteration.

- (1) Halt after a fixed number of iterations: $k = K$.
- (2) l_2 bounded noise: Halt when no column explains a significant amount of energy in the residual: $\|\mathbf{r}_k\|_2 \leq \varepsilon$.
- (3) l_∞ bounded noise: Halt when no column explains a significant amount of energy in the residual: $\|A^* \mathbf{r}_k\|_\infty \leq \varepsilon$ where A^* denotes the transpose of A .

The mutual incoherence property [8] and the restricted isometry property [5] of the measurement matrix have been used for the analysis of OMP. Let A_i be the i th column of the matrix A . In this paper we assume $\|A_i\|_2 = 1$, $i = 1, \dots, n$. The *mutual incoherence* is defined by

$$\mu(A) = \max_{i \neq j} |\langle A_i, A_j \rangle|,$$

A given matrix A satisfies the *restricted isometry property* of order K if there exist a δ_K such that

$$(1 - \delta_K) \|\mathbf{x}\|_2^2 \leq \|A\mathbf{x}\|_2^2 \leq (1 + \delta_K) \|\mathbf{x}\|_2^2 \quad \text{for all} \quad \|\mathbf{x}\|_0 \leq K. \quad (1.3)$$

The smallest constant δ_K is called the *restricted isometry constant*. Many types of random matrices satisfy the RIP with high probability, such as subgaussian random matrix [1] and random partial Fourier matrix [15]. The mutual incoherence property is stronger than the RIP: $\delta_{K+1} \leq K\mu(A)$.

In [16], Tropp has shown $\mu(A) < \frac{1}{2K-1}$ is a sufficient condition for reconstructing any K -sparse signal in the noiseless. Then Cai, Wang and Xu proved this condition is sharp in [2]. In [6], Davenport and Wakin have

showed that there exist matrices satisfying some RIP but not the mutual coherence condition via numerical experiments. This motivated them to establish the RIP-based sufficient conditions. They have proved that the restricted isometry constant $\delta_{3K} < \frac{1}{3\sqrt{K}}$ is sufficient for OMP to recover any K -sparse signal in K steps. Several papers have improved the sufficient condition, such as [10] and [11]. Very recently, Mo and Shen have improved the sufficient condition to

$$\delta_{K+1} < \frac{1}{\sqrt{K} + 1}. \quad (1.4)$$

For any $K \geq 2$, they also constructed a matrix with the restricted isometry constant $\delta_{K+1} = \frac{1}{\sqrt{K}}$ such that OMP can not recover some K -sparse signal \mathbf{x} in K iterations. Hence, the estimate (1.4) is near-optimal.

For the noise case, Cai and Wang have provided coherence-based guarantees for OMP [3]. This subject was also considered in [9] and [18]. However, there are few results on the general model (1.1) by using the RIP. Following the line of [12], we investigate the OMP in the noise case under the RIP-based conditions.

The rest of paper is organized as follows. In section 2, we shall introduce some notations and investigate some properties of the restricted isometry constants. In section 3, the main results are established for OMP recovering the sparse signals with noise.

2 Preliminaries

Before going further, we introduce some notations. Suppose T is a subset of $\{1, \dots, n\}$. Let $T^c = \{1, 2, \dots, n\} \setminus T$. For a given matrix A , denote

$$A_T = \begin{cases} A_i, & i \in T, \\ 0, & \text{otherwise.} \end{cases}$$

For convenience, A_T also denotes the submatrix of A corresponding to T . We use the same way to define \mathbf{x}_T for the vector $\mathbf{x} \in \mathbb{R}^n$. Thus, we have

$$A_T \mathbf{x} = A \mathbf{x}_T = A_T \mathbf{x}_T.$$

The pseudo inverse of a tall, full-rank matrix A is defined by $A^\dagger = (A^* A)^{-1} A$. The support of $\mathbf{x} = \{x_1, \dots, x_n\}$ is denoted by $\text{supp}(\mathbf{x}) = \{i : x_i \neq 0\}$. Let e_i be the i th coordinate unit vector in \mathbb{R}^n . We denote

$$S_i(\mathbf{x}) := \langle A e_i, A \mathbf{x} \rangle, \quad i = 1, \dots, n,$$

Table 1: Orthogonal Matching Pursuit

Input: A, \mathbf{y}
Set: $\Omega_0 = \emptyset, \mathbf{x}_0 = 0, k = 1$ while not converge $\mathbf{r}_k = \mathbf{y} - A_{\Omega_{k-1}} \mathbf{x}_{k-1}$ $\Omega_k = \Omega_{k-1} \cup \arg \max_i \langle A e_i, \mathbf{r}_k \rangle $ $\mathbf{x}_k = (A_{\Omega_k}^* A_{\Omega_k})^{-1} A_{\Omega_k}^* \mathbf{y}$ $k = k + 1$ end while set: $\hat{\mathbf{x}}_{\Omega_k} = \mathbf{x}_k, \hat{\mathbf{x}}_{\Omega_k^c} = 0$
Return: \hat{x}

$$S_T(\mathbf{x}) := \max_{i \in T} |S_i(\mathbf{x})|,$$

and

$$E(\mathbf{z}) := \max_{i \in \{1, \dots, n\}} |\langle A e_i, \mathbf{z} \rangle|.$$

Table 1 shows the framework of OMP.

Now we investigate some properties of the restricted isometry constant. Lemma 2.1 were established by Needell and Tropp in [13].

Lemma 2.1. *Let \mathbf{x} be a K -sparse vector. Suppose the matrix A has the restricted isometry constant δ_K . Then for $T \subset \text{supp}(\mathbf{x})$,*

1. $\|A_T^* A_{T^c} \mathbf{x}\|_2 \leq \delta_K \|\mathbf{x}_{T^c}\|_2.$
2. $\|(A_T^* A_T)^{-1} \mathbf{x}\|_2 \leq \frac{1}{1-\delta_K} \|\mathbf{x}\|_2.$

The following lemma was obtained by Cai and Wang in [3].

Lemma 2.2. *Let \mathbf{x} be a K -sparse vector with $\Omega = \text{supp}(\mathbf{x})$. Suppose that the matrix A has the restricted isometry constant δ_K . Then for $T \subset \Omega$,*

1. $(1 - \delta_K) \|\mathbf{x}_{\Omega \setminus T}\|_2 \leq \|A_{\Omega \setminus T}^* (I - A_T A_T^\dagger) A_{\Omega \setminus T} \mathbf{x}_{\Omega \setminus T}\|_2 \leq (1 + \delta_K) \|\mathbf{x}_{\Omega \setminus T}\|_2.$
2. $(1 - \delta_{K+1}) \|\mathbf{x}_{T^c}\|_2 \leq \|A(I - A_T^\dagger A) \mathbf{x}\|_2.$

Lemma 2.3. *Let \mathbf{x} be a K -sparse vector. Suppose that the matrix A has the restricted isometry constant δ_K . Then for any $T \subset \text{supp}(\mathbf{x})$*

$$\|(I - A_T^\dagger A) \mathbf{x}\|_2 \leq \frac{\|\mathbf{x}_{T^c}\|_2}{1 - \delta_K}. \quad (2.1)$$

Proof. Split $\mathbf{x} = \mathbf{x}_T + \mathbf{x}_{T^c}$, we have

$$\begin{aligned}
(I - A_T^\dagger A)\mathbf{x} &= \mathbf{x}_T + \mathbf{x}_{T^c} - A_T^\dagger A_T \mathbf{x}_T - A_T^\dagger A_{T^c} \mathbf{x}_{T^c} \\
&= \mathbf{x}_T + \mathbf{x}_{T^c} - \mathbf{x}_T - A_T^\dagger A_{T^c} \mathbf{x}_{T^c} \\
&= \mathbf{x}_{T^c} - A_T^\dagger A_{T^c} \mathbf{x}_{T^c}.
\end{aligned}$$

By Lemma 2.1, we get

$$\begin{aligned}
\|(I - A_T^\dagger A)\mathbf{x}\|_2 &\leq \|\mathbf{x}_{T^c}\|_2 + \|A_T^\dagger A_{T^c} \mathbf{x}_{T^c}\|_2 \\
&\leq \|\mathbf{x}_{T^c}\|_2 + \|(A_T^* A_T)^{-1} A_T^* A_{T^c} \mathbf{x}_{T^c}\|_2 \\
&\leq \|\mathbf{x}_{T^c}\|_2 + \frac{\delta_K}{1 - \delta_K} \|\mathbf{x}_{T^c}\|_2 \\
&\leq \frac{\|\mathbf{x}_{T^c}\|_2}{1 - \delta_K}.
\end{aligned}$$

□

3 l_2 Bounded Noise

In this section, we shall prove the main results of the paper. Both the stopping rule 2 and the stopping rule 3 of OMP for the noise case are considered. We first consider the noise \mathbf{z} is bounded by $\|\mathbf{z}\|_2 \leq B_2$. Then the stopping rule is $\|\mathbf{r}_k\|_2 \leq B_2$.

Lemma 3.1. Suppose $\delta_{K+1} < \frac{1}{\sqrt{K+3}}$, we have

$$(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K}) > 0. \quad (3.1)$$

Proof. Simple calculate shows that (3.1) is equal to

$$\frac{1}{\delta_{K+1}} + \delta_{K+1} > \sqrt{K} + 3.$$

Thus, $\delta_{K+1} < \frac{1}{\sqrt{K+3}}$ is the stronger condition. □

The following results is a key tool in this paper.

Theorem 3.2. Assume $\delta_{K+1} < \frac{1}{\sqrt{K+3}}$. For any given K -sparse signal \mathbf{x} . Suppose that the measurement matrix A has the restricted isometry constant δ_{K+1} satisfying

$$\|\mathbf{x}_{\Omega_k^c}\|_2 > \frac{2(1 - \delta_{K+1})E(\mathbf{z}_k)\sqrt{K - k}}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K - k})}. \quad (3.2)$$

where $\mathbf{z}_k = (I - A_{\Omega_k} A_{\Omega_k}^\dagger) \mathbf{z}$. Then OMP selects an index of the support of \mathbf{x} at the $(k+1)$ th iteration.

Proof. For a given K -sparse signal \mathbf{x} , denote the support of \mathbf{x} by Ω . Consider the $(k+1)$ -th iteration,

$$\begin{aligned} \mathbf{r}_{k+1} &= \mathbf{y} - A_{\Omega_k} \mathbf{x}_k \\ &= A\mathbf{x} + \mathbf{z} - A_{\Omega_k} A_{\Omega_k}^\dagger (A\mathbf{x} + \mathbf{z}) \\ &= A(I - A_{\Omega_k}^\dagger A)\mathbf{x} + (I - A_{\Omega_k} A_{\Omega_k}^\dagger) \mathbf{z}. \end{aligned}$$

For simplify, let $\mathbf{t}_k = (I - A_{\Omega_k}^\dagger A)\mathbf{x}$. Then we get

$$\begin{aligned} \langle Ae_i, \mathbf{r}_{k+1} \rangle &= \langle Ae_i, \mathbf{y} - A_{\Omega_k} \mathbf{x}_k \rangle \\ &= \langle Ae_i, A\mathbf{t}_k + \mathbf{z}_k \rangle \\ &= S_i(\mathbf{t}_k) + \langle Ae_i, \mathbf{z}_k \rangle. \end{aligned}$$

Note that the residual \mathbf{r}_k are orthogonal to all the selected columns of A , so no index is selected twice. Thus, the sufficient condition for choosing an index from $\Omega \setminus \Omega_k$ in the $(k+1)$ th iteration is

$$S_{\Omega \setminus \Omega_k}(\mathbf{t}_k) - E(\mathbf{z}_k) > |S_i(\mathbf{t}_k)| + E(\mathbf{z}_k) \quad \text{for all } i \in \Omega^c. \quad (3.3)$$

In the rest of the proof, we shall give a sufficient condition for (3.3) holds.

Note the support of \mathbf{t}_k is a subset of Ω . By Lemma 2.1 in [4], we have

$$|S_i(\mathbf{t}_k)| = |\langle Ae_i, A\mathbf{t}_k \rangle| \leq \delta_{K+1} \|\mathbf{t}_k\|_2 \quad \text{for all } i \in \Omega^c. \quad (3.4)$$

Combine (2.1) and (3.4) leads to

$$|S_i(\mathbf{t}_k)| \leq \frac{\delta_{K+1}}{1 - \delta_{K+1}} \|\mathbf{x}_{\Omega_k^c}\|_2, \quad \text{for all } i \in \Omega^c. \quad (3.5)$$

By Lemma 2.2, we obtain

$$S_{\Omega \setminus \Omega_k}(\mathbf{t}_k) \geq \frac{\|A_{\Omega \setminus \Omega_k}^* A \mathbf{t}_k\|_2}{\sqrt{K-k}} \geq \frac{(1 - \delta_K) \|\mathbf{x}_{\Omega_k^c}\|_2}{\sqrt{K-k}}. \quad (3.6)$$

It follows from (3.5) and (3.6) that the sufficient condition for (3.3) holds is

$$\frac{(1 - \delta_K) \|\mathbf{x}_{\Omega_k^c}\|_2}{\sqrt{K-k}} - \frac{\delta_{K+1}}{1 - \delta_{K+1}} \|\mathbf{x}_{\Omega_k^c}\|_2 > 2E(\mathbf{z}_k)$$

which is simplified to (3.2). \square

Theorem 3.3. Suppose $\|\mathbf{z}\|_2 < B_2$ and $\delta_{K+1} < \frac{1}{\sqrt{K+3}}$. Then OMP with the stopping rule $\|\mathbf{r}_k\|_2 \leq B_2$ finds the support of \mathbf{x} if all the nonzero coefficients x_i satisfy

$$|x_i| > \frac{2(1 - \delta_{K+1})B_2}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K})}. \quad (3.7)$$

Proof. We first estimate the $E(\mathbf{z}_k)$. Since $\|\mathbf{z}\|_2 \leq B_2$, we have

$$|\langle Ae_i, \mathbf{z}_k \rangle| \leq \|A_i\|_2 \|(I - A_{\Omega_k} A_{\Omega_k}^\dagger) \mathbf{z}\|_2 \leq \|\mathbf{z}\|_2 \leq B_2.$$

This implies

$$E(\mathbf{z}_k) \leq B_2.$$

Hence, (3.2) is followed by

$$\|\mathbf{x}_{\Omega_k^c}\|_2 > \frac{2(1 - \delta_{K+1})B_2\sqrt{K - k}}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K - k})}. \quad (3.8)$$

Since

$$\frac{\|\mathbf{x}_{\Omega_k^c}\|_2}{\sqrt{K - k}} \geq \min_{i \in \Omega_k^c} |x_i|,$$

(3.8) is implied by

$$|x_i| > \frac{2(1 - \delta_{K+1})B_2}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K})}, \quad \text{for all } i \in \Omega_k^c.$$

Now we prove that the OMP do not stop for some $j + 1 < k$. Consider the $j + 1$ iteration for some $j + 1 < k$, by Lemma 2.2, we have

$$\begin{aligned} \|\mathbf{r}_{j+1}\|_2 &= \|\mathbf{y} - A_{\Omega_j} \mathbf{x}_j\|_2 \\ &= \|A(I - A_{\Omega_j}^\dagger A) \mathbf{x} + (I - A_{\Omega_j} A_{\Omega_j}^\dagger) \mathbf{z}\|_2 \\ &\geq \|A(I - A_{\Omega_j}^\dagger A) \mathbf{x}\|_2 - \|(I - A_{\Omega_j} A_{\Omega_j}^\dagger) \mathbf{z}\|_2 \\ &> (1 - \delta_{K+1}) \|\mathbf{x}_{\Omega_j^c}\|_2 - B_2. \end{aligned} \quad (3.9)$$

By (3.7), we obtain

$$(1 - \delta_{K+1}) \|\mathbf{x}_{\Omega_j^c}\|_2 \geq \frac{2(1 - \delta_{K+1})^2 B_2}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K})} \geq 2B_2. \quad (3.10)$$

It follows from (3.9) and (3.10) that $\|\mathbf{r}_{j+1}\|_2 > B_2$. \square

We assume that \mathbf{z} is zero-mean white Gaussian noise with covariance $\sigma^2 I_{m \times m}$. Cai, Xu and Zhang have show that $z \sim N(0, \sigma^2 I_{m \times m})$ satisfies

$$P(\mathbf{z} \in B_2) \geq 1 - 1/m$$

where $B_2 = \{\mathbf{z} : \|\mathbf{z}\|_2 \leq \sigma \sqrt{m + 2\sqrt{m \log m}}\}$. With this argument and Theorem 3.3, we obtain the following result.

Theorem 3.4. *Suppose $z \sim N(0, \sigma^2 I_{m \times m})$, $\delta_{K+1} < \frac{1}{\sqrt{K+3}}$ and nonzero coefficients x_i satisfy*

$$|x_i| > \frac{2(1 - \delta_{K+1})\sigma \sqrt{m + 2\sqrt{m \log m}}}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K})}.$$

Then OMP with the stopping rule $\|\mathbf{r}_k\|_2 \leq \sigma \sqrt{m + 2\sqrt{m \log m}}$ finds the support of \mathbf{x} with probability at least $1 - 1/m$.

Now we give the RIP-based sufficient conditions for OMP with l_∞ bounded noise case. Then the stopping rule is $\|A^* \mathbf{r}_k\|_\infty \leq B_\infty$.

Theorem 3.5. *Suppose $\|A^* \mathbf{z}\|_\infty < B_\infty$ and $\delta_{K+1} < \frac{1}{\sqrt{K+3}}$. Then OMP with the stopping rule $\|A^* \mathbf{r}\|_\infty < B_\infty$ finds the support of \mathbf{x} if all the nonzero coefficients x_i satisfy*

$$|x_i| > \frac{2(1 - \delta_{K+1})B_\infty}{(1 - \delta_{K+1})^2 - \delta_{K+1}(1 + \sqrt{K})} \left(1 + \frac{\sqrt{K}}{\sqrt{1 - \delta_{K+1}}}\right). \quad (3.11)$$

Proof. Since the proof is similar as the proof of Theorem 4 in [3]. We include a sketch for the completeness. To make sure (3.2) of Theorem 3.2 hold, we first give an estimation of $E(z_k)$ in the $(k+1)$ -th iteration. We have

$$|\langle Ae_i, \mathbf{z}_k \rangle| \leq |A_i^* \mathbf{z}| + |\langle Ae_i, A_{\Omega_k} A_{\Omega_k}^\dagger \mathbf{z} \rangle| \leq B_\infty \left(1 + \frac{\sqrt{k}}{\sqrt{1 - \delta_{K+1}}}\right).$$

Together with (3.11), it implies that (3.2) holds. Now consider the t -th iteration with $t < k+1$. We obtain

$$\begin{aligned} \|A^* z_t\|_\infty &\geq \frac{1 - \delta_{K+1}}{\sqrt{K} - t} \|\mathbf{x}_{\Omega_t}\|_2 - \left(1 + \frac{\sqrt{t}}{\sqrt{1 - \delta_{K+1}}}\right) B_\infty \\ &\geq 2 \left(1 + \frac{\sqrt{K}}{\sqrt{1 - \delta_{K+1}}}\right) B_\infty - \left(1 + \frac{\sqrt{t}}{\sqrt{1 - \delta_{K+1}}}\right) B_\infty \\ &\geq B_\infty. \end{aligned}$$

The second inequality is implied by (3.11). Therefore, the OMP does not stop after t -th iteration.

□

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